FLUID MECHANICS

DEFINITION OF FLUID

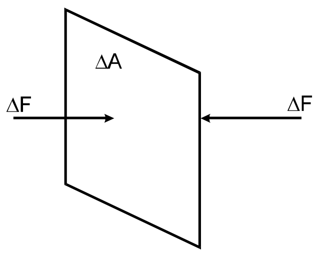
The term fluid refers to a substance that can flow and does not have a shape of its own. For example liquid and gases.

Fluid includes property → (A) Density (B) Viscosity (C) Bulk modulus of elasticity (D) pressure (E) specific gravity

PRESSURE IN A FLUID

The pressure p is defined as the magnitude of the normal force acting on a unit surface area.

$P = \frac{\Delta F}{\Delta A}$ ΔF = normal force on a surface area ΔA.



The pressure is a scalar quantity. This is because hydrostatic pressure is transmitted equally in all directions when force is applied, which shows that a definite direction is not associated with pressure.

Thrust. The total force exerted by a liquid on any surface in contact with it is called thrust of the liquid.

CONSEQUENCES OF PRESSURE

(i) Railway tracks are laid on large sized wooden or iron sleepers. This is because the weight (force) of the train is spread over a large area of the sleeper. This reduces the pressure acting on the ground and hence prevents the yielding of ground under the weight of the train.

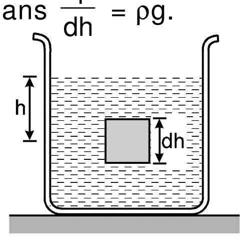
(ii) A sharp knife is more effective in cutting the objects than a blunt knife.
The pressure exerted = Force/area. The sharp knife transmits force over a small area as compared to the blunt knife. Hence the pressure exerted in case of sharp knife is more than in case of blunt knife.

(iii) A camel walks easily on sand but a man cannot inspite of the fact that a camel is much heavier than man.
This is because the area of camel's feet is large as compared to man's feet. So the pressure exerted by camel on the sand is very small as compared to the pressure exerted by man. Due to large pressure, sand under the feet of man yields and hence he cannot walk easily on sand.

VARIATION OF PRESSURE WITH HEIGHT

Weight of the small element dh is balanced by the excess pressure. It means $\frac{dp}{dh} = \rho g$.

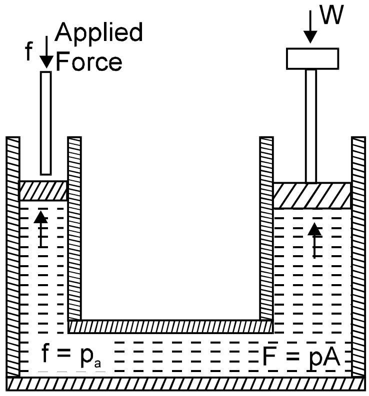
$$\int_{P_a}^{P} dp = \rho g \int_{0}^{h} dh \quad \Rightarrow \quad P = P_a + \rho g h$$



PASCAL'S LAW

if the pressure in a liquid is changed at a particular, point the change is transmitted to the entire liquid without being diminished in magnitude. In the above case if P_a is increased by some amount than P must increase to maintained the difference $(P - P_a) = h\rho g$. This is Pascal's Law which states that Hydraulic lift is common application of Pascal's Law.

1. Hydraulic press.

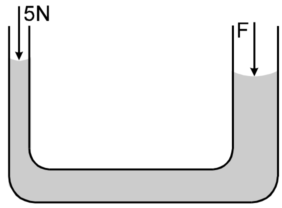


$$p = \frac{f}{a} = \frac{W}{A} \text{ or } f = \frac{W}{A} \times a$$

as A >> a then f << W..

This can be used to lift a heavy load placed on the platform of larger piston or to press the things placed between the piston and the heavy platform. The work done by applied force is equal to change in potential energy of the weight in hydraulic press.

Ex.1 The area of cross-section of the two arms of a hydraulic press are 1 cm^2 and 10 cm^2 respectively (figure). A force of 5 N is applied on the water in the thinner arm. What force should be applied on the water in the thicker arms so that the water may remain in equilibrium?



Sol. In equilibrium, the pressures at the two surfaces should be equal as they lie in the same horizontal level. If the atmospheric pressure is P and a force F is applied to maintain the equilibrium, the pressures are

$$P_0 + \frac{5\,N}{1\,cm^2} \text{ and } P_0 + \frac{F}{10\,cm^2} \text{ respectively.}$$

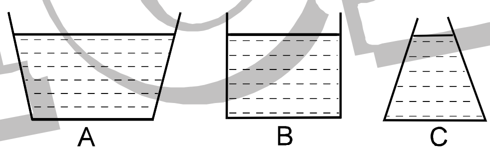
This givens F = 50 N.

2. Hydraulic Brake.

Hydraulic brake system is used in auto-mobiles to retard the motion.

HYDROSTATIC PARADOX

Pressure is directly proportional to depth and by applying pascal's law it can be seen that pressure is independent of the size and shape of the containing vessel.



$P_A = P_B = P_C$

ATMOSPHERIC PRESSURE

Definition.

The atmospheric pressure at any point is numerically equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere.

At 0ºC, density of mercury = 13.595 g cm^{-3}, and at sea level, g = 980.66 cm s^{-2}

Now P = hρg.

Atmospheric pressure = 76 × 13.595 × 980.66 dyne cm^{-2} = 1.013 × 10^{-5} N-m^2 (p_a)

Height of Atmosphere

The standard atmospheric pressure is 1.013 × 10^5 Pa (N m^{-2}). If the atmosphere of earth has a uniform density ρ = 1.30 kg m^{-3}, then the height h of the air column which exert the standard atmospheric pressure is given by

⇒ $h\rho g = 1.013 \times 10^5$

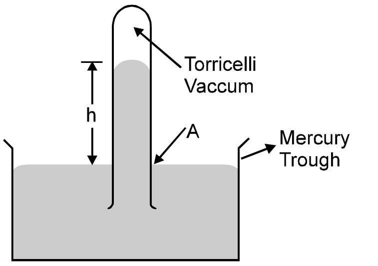
$$h = \frac{1.013 \times 10^5}{\rho g} = \frac{1.013 \times 10^5}{1.13 \times 9.8} \text{ m} = 7.95 \times 10^3 \text{ m} \sim 8 \text{ km.}$$

In fact, density of air is not constant but decreases with height. The density becomes half at about 6 km high, $\frac{1}{4}$th at about 12 km and so on. Therefore, we can not draw a clear cut line above which there is no atmosphere. Anyhow the atmosphere extends upto 1200 km. This limit is considered for all practical purposes.

MEASUREMENT OF ATMOSPHERIC PRESSURE

1. Mercury Barometer.

To measure the atmospheric pressure experimentally, torricelli invented a mercury barometer in 1643.



$$p_a = h\rho g$$

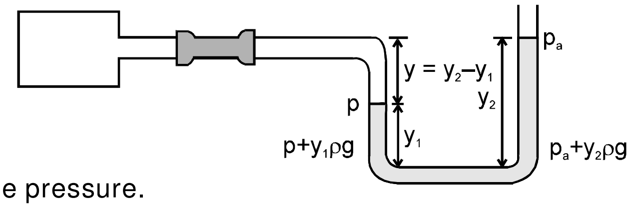
The pressure exerted by a mercury column of 1mm high is called 1 Torr.

1 Torr = 1 mm of mercury column

2. Open tube Manometer

Open-tube manometer is used to measure the pressure gauge.

When equilibrium is reached, the pressure at the bottom of left limb is equal to the pressure at the bottom of right limb.



i.e. $p + y_1 \rho g = p_a + y_2 \rho g$

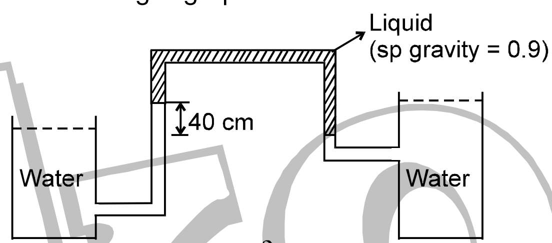
$p - p_a = \rho g\ (y_2 - y_1) = \rho g y$

$p - p_a = \rho g\ (y_2 - y_1) = \rho g y$

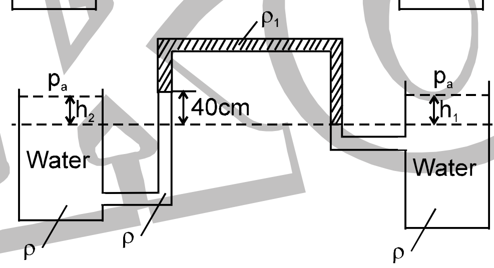
p = absolute pressure, $p - p_a$ = gauge pressure.

Thus, knowing y and ρ (density of liquid), we can measure the gauge pressure.

Ex. 2 The manometer shown below is used to measure the difference in water level between the two tanks. Calculate this difference for the conditions indicated.



Ans. 4 cm



Sol. $p_a + h_1 \rho g - 40\rho_1 g + 40\rho g = p_a + h_2 \rho g$

$h_2 \rho g - h_1 \rho g = 40\ \rho g - 40\ \rho_1 g$

as $\rho_1 = 0.9\rho$

$(h_2 - h_1)\ \rho g = 40\rho g - 36\rho g$

$h_2 - h_1 = 4$ cm

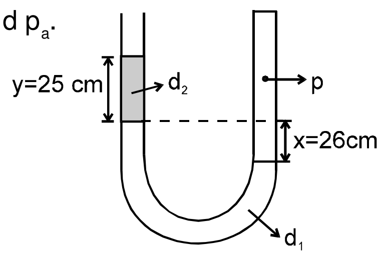
3. Water Barometer.

Let us suppose water is used in the barometer instead of mercury.

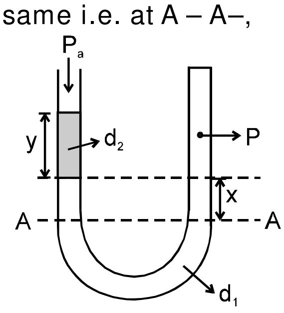
$$h\rho g = 1.013 \times 10^5 \qquad \text{or} \qquad h = \frac{1.013 \times 10^5}{\rho g}$$

The height of the water column in the tube will be 10.3 m. Such a long tube cannot be managed easily, thus water barometer is not feasible.

Ex. 3 In a given U-tube (open at one-end) find out relation between p and p_a.
Given $d_2 = 2 \times 13.6$ gm/cm³ $d_1 = 13.6$ gm/cm³



Sol. Pressure in a liquid at same level is same i.e. at A – A–,



$$p_a + d_2 y g + x d_1 g = p$$

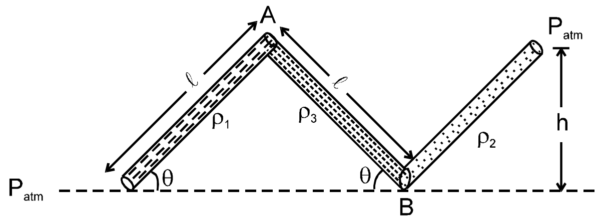
In C.G.S.

$p_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = p$

$p_a + 13.6 \times g\,[50 + 26] = p$

$2p_a = p \qquad [p_a = 13.6 \times g \times 76]$

Ex. 4 Find out pressure at points A and B. Also find angle 'θ'.



Sol. Pressure at A – $\quad P_A = P_{atm} - \rho_1\, gl \sin\theta$

Pressure at B $\quad P_B = P_{atm} + \rho_2\, gh\ \theta$

But P_B is also equal to $\quad P_B = P_A + \rho_3\, gl \sin\theta$

Hence - $\quad P_{atm} + \rho_2\, gh = P_A + \rho_3\, gl \sin\theta$

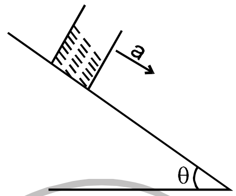
$P_{atm} + \rho_2\, gh = P_{atm} - \rho_1\, gl \sin\theta + \rho_3\, gl \sin\theta$

$$\sin\theta = \frac{\rho_2 h}{(\rho_3 - \rho_1)\ell}.$$

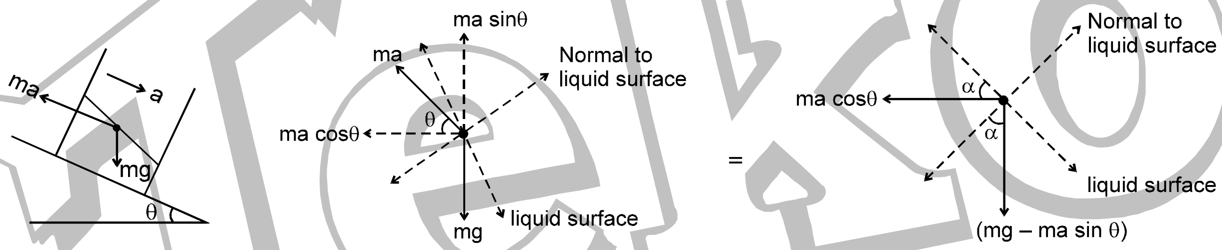
Ex. 5 In the given figure, the container slides down with acceleration 'a' on an incline of angle 'θ'. Liquid is stationary with respect to container. Find out -

(i) Angle made by surface of liquid with horizontal plane.

(ii) Angle if $a = g \sin\theta$.



Sol. Consider a fluid particle on surface. The forces acting on it are shown in figure.



Resultant force acting on liquid surface, will always normal to it

$$\tan\alpha = \frac{ma\cos\theta}{mg - ma\sin\theta} = \frac{a\cos\theta}{(g - a\sin\theta)}$$

Thus angle of liquid surface with the horizontal is equal to $\quad \alpha = \tan^{-1}\dfrac{a\cos\theta}{(g - a\sin\theta)}$

(ii) If $a = g \sin\theta$, then $\quad \alpha = \tan^{-1}\left(\dfrac{a\cos\theta}{g - g\sin^2\theta}\right) = \tan^{-1}\dfrac{g\sin\theta\cos\theta}{g\cos^2\theta}$

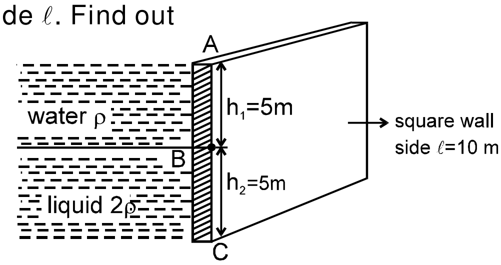
$= \tan^{-1}(\tan\theta) \qquad \alpha = \theta$

Ex. 6 Water and liquid is filled up behind a square wall of side ℓ. Find out

(a) Pressures at A, B and C

(b) Forces in part AB and BC

(c) Total force and point of application of force. (neglect atmosphere pressure in every calculation)



Sol. (a) As there is no liquid above 'A',

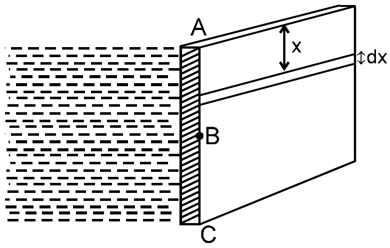
So pressure at A, $p_A = 0$

Pressure at B, $p_B = \rho g h_1$

Pressure at C, $p_C = \rho g h_1 + 2\rho g h_2$

(b) Force at A = 0

Take a strip of width 'dx' at a depth 'x' in part AB.



Pressure is equal to $\rho g x$.

Force on strip = pressure × area

$dF = \rho g x \ \ell dx$

Total force upto B

$$F = \int_0^{h_1} \rho g x \ell dx = \frac{\rho g x \ell h_1^2}{2} = \frac{1000 \times 10 \times 10 \times 5 \times 5}{2}$$

$= 1.25 \times 10^6$ N

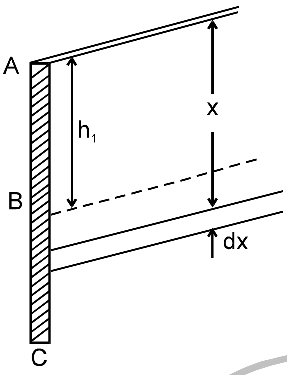
In part BC for force take a elementary strip of width dx in portion BC. Pressure is equal to

$= \rho g h_1 + 2\rho g(x - h_1)$

Force on elementary strip = pressure × area

$dF = [\rho g h_1 + 2\rho g(x - h_1)] \ \ell \ dx$

Total force on part BC



$$F = \int_{h_1}^{\ell} [\rho g h_1 + 2\rho g(x - h_1)] \ell dx$$

$$= \left[\rho g h_1 x + 2\rho g \left[\frac{x^2}{2} - h_1 x \right] \right]_{h_1}^{\ell} \ell$$

$$= \rho g h_1 h_2 \ \ell + 2\rho g \ell \left[\frac{\ell^2 - h_1^2}{2} - h_1 \ell + h_1^2 \right]$$

$= \rho g h_1 h_2 \ell + \dfrac{2\rho g \ell}{2} \ [\ell^2 + h_1^2 - 2h_1 \ell] \qquad = \rho g h_1 h_2 \ell + \rho g \ell \ (\ell - h_1)^2$

$= \rho g h_2 \ell \ [h_1 + h_2] = \rho g h_2 \ell^2 \qquad = 1000 \times 10 \times 5 \times 10 \times 10 = 5 \times 10^6$ N

(c) Total force $= 5 \times 10^6 + 1.25 \times 10^6 \qquad = 6.25 \times 10^6$ N

Taking torque about A

Total torque of force in AB $= \int dF \cdot x \qquad = \int_0^{h_1} \rho g x \ell dx . x$

$$= \left[\frac{\rho g \ell x^3}{3} \right]_0^{h_1} = \frac{\rho g \ell h_1^3}{3} = \frac{1000 \times 10 \times 10 \times 125}{3} = \frac{1.25 \times 10^7}{3} \text{ N - m}$$

Total torque of force in BC $= \int dF \cdot x$

On solving we get $= \rho g h_1 h_2 \ell [h_1 + \dfrac{h_2}{2}] + \rho g h_2^2 \ell [h_1 + \dfrac{2h_2}{3}]$

$= 1000 \times 10 \times 5 \times 5 \times 10 \ [5 + 2.5] + 1000 \times 10 \times 25 \times 10 \ [5 + \dfrac{10}{3}]$

$= 2.5 \times 7.5 \times 10^6 + \dfrac{62.5}{3} \times 10^6 \qquad = \dfrac{118.75}{3} \times 10^6$

Total torque $= \dfrac{11.875 \times 10^7}{3} + \dfrac{1.25 \times 10^7}{3} = \dfrac{13.125 \times 10^7}{3}$

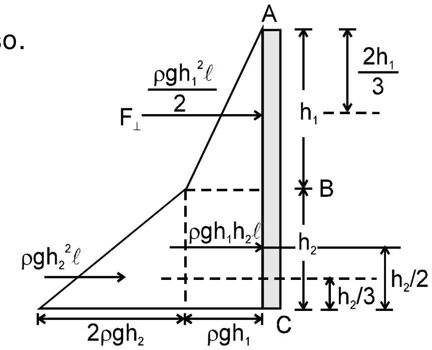
Total torque = total force × distance of point of application of force from top

$= F \ . \ x_p$

$6.25 \times 10^6 \ x_p = \dfrac{13.125 \times 10^7}{3}$

$x_p = 7$m

Alternatively



We can solve this problem by pressure diagram also.
Force on 'AB' part is area of triangle 'ABC'

$$F_{AB} = \rho g h_1 \times \frac{h_1}{2} \times \ell \qquad = \frac{\rho g h_1^2 \ell^2}{2}$$

Torque of force of AB part about A -

$$\tau_{AB} = \frac{\rho g h_1^2 \ell}{2} \times \frac{2h_1}{3}$$

$$= \frac{\rho g h_1^3 \ell}{3} = \frac{\rho g \ell^4}{24}$$

Force on 'BC' part is area of trapezium -

$$F_{BC} = \rho g h_1 h_2 \ell + 2\rho g h_2 \times \frac{h_2}{2} \ell \qquad = \rho g h_1 h_2 \ell + \rho g h_2{}^2 \ell$$

Torque of force of 'BC' part about 'A' -

$$\tau_{BC} = \rho g h_1 h_2 \ell \ \left(h_1 + \frac{h_2}{2}\right) + \rho g h_2{}^2 \ell \left(h_1 + \frac{2h_2}{3}\right)$$

$$= \frac{\rho g \ell^3}{4}\left[\frac{\ell}{2} + \frac{\ell}{4}\right] + \rho g \frac{\ell^3}{4}\left[\frac{\ell}{2} + \frac{\ell}{3}\right]$$

$$= \frac{\rho g \ell^3}{4}\left[\ell + \frac{\ell}{3} + \frac{\ell}{4}\right] = 19\frac{\rho g \ell^4}{48}$$

$$\text{Total force} = \frac{\rho g h_1^2 \ell}{2} + \rho g h_1 h_2 \ell + \rho g h_2{}^2 \ell$$

$$= \frac{\rho g \ell^3}{8} + \rho g \frac{\ell^3}{4} + \frac{\rho g \ell^3}{4}\left[1 + 1 + \frac{1}{2}\right] \qquad = \frac{5\rho g \ell^3}{8}$$

$$\text{Total torque} = \frac{19\rho g \ell^4}{48} + \frac{\rho g \ell^4}{24} = \frac{21\rho g \ell^4}{48}$$

But $\quad F\,x_p = \dfrac{21\rho g \ell^4}{48} \qquad \dfrac{5\rho g \ell^3}{8} \times p = \dfrac{21\rho g \ell^4}{48}$

$$x_p = \frac{21\ell}{30} = \frac{21 \times 10}{30} = 7 \text{ m}$$

Thus total force is acting at 7m below A point.

ARCHIMEDES' PRINCIPLE

According to this principle, when a body is immersed wholly or partially in a fluid, it loses its weight which is equal to the weight of the fluid displaced by the body.

Up thrust = buoyancy = $V\rho_\ell g$ $\qquad$ V = volume submerged $\qquad$ ρ_ℓ = density of liquid.

Relation between density of solid and liquid

weight of the floating solid = weight of the liquid displaced

$$V_1\,\rho_1\,g = V_2\,\rho_2\,g \qquad \Rightarrow \qquad \frac{\rho_1}{\rho_2} = \frac{V_2}{V_1}$$

or $\quad \dfrac{\text{Density of solid}}{\text{Density of liquid}} = \dfrac{\text{Volume of the immeresed portion of the solid}}{\text{Total Volume of the solid}}$

This relationship is valid in accelerating fluid also. Thus, the force acting on the body are :
(i) its weight Mg which acts downward and
(ii) net upward thrust on the body or the buoyant force (mg)
Hence the apparent weight of the body = Mg – mg = weight of the body – weight of the displaced liquid.
Or Actual Weight of body – Apparent weight of body = weight of the liquid displaced.
The point through which the upward thrust or the buoyant force acts when the body is immersed

in the liquid is called its centre of buoyancy. This will coincide with the centre of gravity if the solid body is homogeneous. On the other hand if the body is not homogeneous, then the centre of gravity may not lie on the line of the upward thrust and hence there may be a torque that causes rotation in the body.

If the centre of gravity of the body and the centre of buoyancy lie on the same straight line, the body is in equilibrium.

If the centre of gravity of the body does not coincide with the centre of buoyancy (i.e., the line of upthrust), then torque acts on the body. This torque causes the rotational motion of the body.

Ex. 7 A copper piece of mass 10 g is suspended by a vertical spring. The spring elongates 1 cm over its natural length to keep the piece in equilibrium. A beaker containing water is now placed below the piece so as to immerse the piece completely in water. Find the elongation of the spring. Density of copper = 9000 kg/m^3. Take g = 10 m/s^2.

Sol. Let the spring constant be k. When the piece is hanging in air, the equilibrium condition gives

$k\,(1\text{ cm}) = (0.01\text{ kg})\,(10\text{ m/s})$

or $k\,(1\text{ cm}) = 0.1\text{ N}.$ ....(i)

The volume of the copper piece

$$= \frac{0.01\text{kg}}{9000\,\text{kg/m}^3} = \frac{1}{9} \times 10^{-5}\text{ m}^3.$$

This is also the volume of water displaced when the piece is immersed in water. The force of buoyancy

= weight of the liquid displaced

$$= \frac{1}{9} \times 10^{-5}\text{ m}^3 \times (1000\text{ kg/m}^3) \times (10\text{ m/s}^2) = 0.011\text{ N}.$$

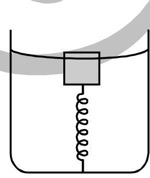
If the elongation of the spring is x when the piece is immersed in water, the equilibrium condition of the piece gives,

$kx = 0.1\text{ N} - 0.011\text{ N} = 0.089\text{ N}.$ ....(ii)

By (i) and (ii),

$$x = \frac{0.089}{0.1}\text{ cm} = 0.89\text{ cm}.$$

Ex. 8 A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of wood = 800 kg/m^3 and spring constant of the spring = 50 N/m. Take g = 10 m/s^2.



Sol. The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside ater = 3 cm – 2.4 = 0.6 cm. Suppose the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water

= volume of the block = $27 \times 10^{-6}\text{ m}^3$.

Hence, the force of buoyancy

$= (27 \times 10^{-6}\text{ m}^3) \times 1(1000\text{ kg/m3}) \times (10\text{ m/s}^2) = 0.27\text{ N}.$

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring

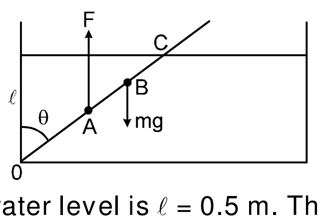
$= 50\text{ N/m} \times 0.6\text{ cm} = 0.3\text{ N}.$

The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is

$W' = (27 \times 10^{-6}\text{ m}) \times (800\text{ kg/m}^3) \times (10\text{ m/s}^2) = 0.22\text{ N}.$

Thus, $W = 0.27\text{ N} + 0.3\text{ N} - 0.22\text{ N} = 0.35\text{ N}.$

Ex. 9 A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position.
(Exclude the case θ = 0).



Sol. The forces acting on the plank are shown in the figure. The height of water level is $\ell = 0.5$ m. The length of the plank is 1.0 m = 2ℓ. The weight of the plank acts through the centre B of the plank. We have OB = ℓ. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have $OA = \frac{OC}{2} = \frac{\ell}{2\cos\theta}$.

Let the mass per unit length of the plank be ρ.
Its weight $mg = 2\ell\rho g$.

The mass of the part OC of the plank $= \left(\frac{\ell}{\cos\theta}\right)\rho$.

The mass of water displaced $= \frac{1}{0.5}\frac{\ell}{\cos\theta}\rho = \frac{2\ell\rho}{\cos\theta}$.

The buoyant force F is, therefore, $F = \frac{2\ell\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.
So, $mg\,(OB)\sin\theta = F(OA)\sin\theta$

or, $(2\ell\rho)\ell = \left(\frac{2\ell\rho}{\cos\theta}\right)\left(\frac{\ell}{2\cos\theta}\right)$

or, $\cos^2\theta = \frac{1}{2}$ or, $\cos\theta = \frac{1}{\sqrt{2}}$, or, $\theta = 45°$.

Ex. 10 A cylindrical block of wood of mass M is floating in water with its axis vertical. It is depressed a little and then released. Show that the motion of the block is simple harmonic and find its frequency.

Sol. Suppose a height h of the block is dipped in the water in equilibrium position. If r be the radius of the cylindrical block, the volume of the water diplaced $= \pi r^2 h$. For floating in equilibrium,

$\pi r^2 h\rho g = W$ ....(i)

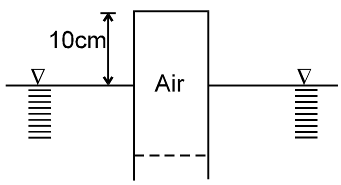
where ρ is the density of water and W the weight of the block.
Now suppose during the vertical motion, the block is further dipped through a distance x at some instant. The volume of the displaced water is $\pi r^2 (h + x)$. The forces acting on the block are, the weight W vertically downward and the buoyancy $\pi r^2(h + x)\rho g$ vertically upward.
Net force on the block at displacement x from the equilibrium position is

$F = W - \pi r^2 (h + x)\rho g = W - \pi r^2 h\rho g - \pi r^2\rho x g$

Using (i) $F = -\pi r^2 \rho g x = -kx$, where $k = \pi r^2\rho g$.
Thus, the block executes SHM with frequency.

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{\pi r^2\rho g}{M}}.$$

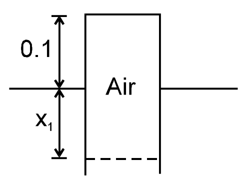
Ex. 11 A cylindrical bucket with one end open is observed to be floating on a water ($\rho = 1000$ kg/m^3) with open and down. It is of 10 N weight and is supported by air that is trapped inside it as shown below. The bucket floats with a height 10 cm above the liquid surface. If the air trapped is assumed to follow isothermal law, then determine the force F necessary just to submerge the bucket. The internal area of cross-section of bucket is 21 cm^2. The thickness of the wall is assumed to negligible and the atmospheric pressure must be neglected. (g = 10 m/sec^2)



Sol. Weight of bucket

$W = Ax_1\rho g$ .....(1)

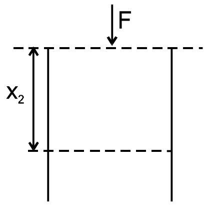
pressure at liquid - air interface = pressure of air $= \rho g x_1$



From (1) $p_1 = \rho g x_1 = \rho g \frac{W}{A\rho g} = \frac{W}{A}$

$v_1 = A[h + x_1] = A\left[h + \frac{W}{A\rho g}\right]$

Let force F is applied



downward force = F + W = Buoyant = $Ax_2 \rho g$ ....(2)

$p_2 = x_2 \rho g$, $v_2 = Ax_2$
$p_1 v_1 = p_2 v_2$

$$\frac{W}{A} \times A\left[h + \frac{W}{A\rho g}\right] = x_2\, \rho\, g\, A\, x_2 \qquad \Rightarrow \qquad x_2 = \sqrt{\frac{W}{A\rho g}\left[h + \frac{W}{A\rho g}\right]}$$

from (2)

$$F + W = A\rho g\sqrt{\frac{W}{A\rho g}\left[h + \frac{W}{A\rho g}\right]} \qquad \Rightarrow \qquad F + W = \sqrt{WA\rho gh + W^2}$$

$$F = \sqrt{WA\rho gh + W^2} - W$$

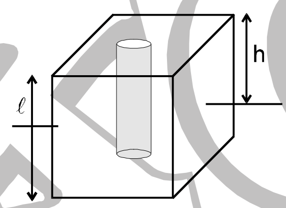
substituting values -

$W = 10$ N, $\rho = 1000$ kg/m^3 , $A = 2.1 \times 10^{-3}$ m^2

$$F = \sqrt{10 \times 2.1 \times 10^{-3} \times 1000 \times 10 \times 10^{-1} + 100} - 10 \quad = 11 - 10 = 1\text{N}$$

Ex.12 A large block of ice cuboid of height 'ℓ' and density $\rho_{ice} = 0.9\, \rho_w$, has a large vertical hole along its axis. This block is floating in a lake. Find out the length of the rope required to raise a bucket of water through the hole.

Sol. Let area of ice-cuboid excluding hole = A
weight of ice block = weight of liquid displaced
$A\, \rho_{ice}\, \ell\, g = A\, \rho_w\, (\ell - h)\, g$



$$\frac{9\ell}{10} = \ell - h \qquad \Rightarrow \qquad h = \ell - \frac{9\ell}{10} = \left(\frac{\ell}{10}\right)$$

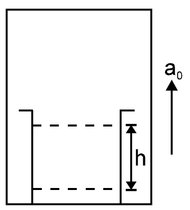
PRESSURE IN CASE OF ACCELERATING FLUID

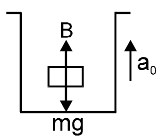
(i) Liquid Placed in elevator :

When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth 'h' may be given by,

$p = h\rho\,[g + a_0]$

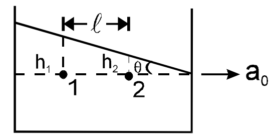
and force of buoyancy, $B = m\,(g + a_0)$





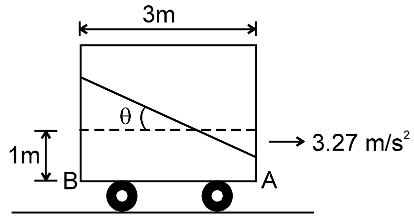
(ii) Free surface of liquid in horizontal acceleration :

$$\tan\theta = \frac{a_0}{g}$$



$p_1 - p_2 = \ell\rho\, a_0$ where p_1 and p_2 are pressures at point 1 & 2. Then $h_1 - h_2 = \dfrac{\ell a_0}{g}$

Ex. 13 An open rectangular tank 1.5 m wide 2m deep and 2m long is half filled with water. It is accelerated horizontally at 3.27 m/sec^2 in the direction of its length. Determine the depth of water at each end of tank. [g = 9.81 m/sec^2]



Sol. $\tan\theta = \frac{a}{g} = \frac{1}{3}$

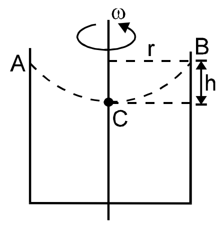
Depth at corner 'A'

$= 1 - 1.5 \tan\theta$

$= 0.5$ m **Ans.**

Depth at corner 'B'

$= 1 + 1.5 \tan\theta = 1.5$ m **Ans.**



(iii) Free surface of liquid in case of rotating cylinder.

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$

STREAMLINE FLOW

The path taken by a particle in flowing fluid is called its line of flow. In the case of steady flow all the particles passing through a given point follow the same path and hence we have a unique line of flow passing through a given point which is also called streamline.

CHARACTERISTICS OF STREAMLINE

1. A tangent at any point on the stream line gives the direction of the velocity of the fluid particle at that point.
2. Two steamlines never intersect each other.

Laminar flow : If the liquid flows over a horizontal surface in the form of layers of different velocities, then the flow of liquid is called Laminar flow. The particle of one layer do not go to another layer. In general, Laminar flow is a streamline flow.

Turbulent Flow : The flow of fluid in which velocity of all particles crossing a given point is not same and the motion of the fluid becomes disorderly or irregular is called turbulent flow.

REYNOLD'S NUMBER

According to Reynold, the critical velocity (v_c) of a liquid flowing through a long narrow tube is
(i) directly proportional to the coefficient of viscosity (η) of the liquid.
(ii) inversely proportional to the density ρ of the liquid and
(iii) inversely proportional to the diameter (D) of the tube.

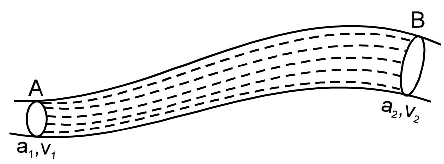
That is $v_c \propto \frac{\eta}{\rho D}$ or $v_c = \frac{R\eta}{\rho D}$ or $= \frac{v_c \rho D}{\eta}$ ....(1)

where R is the Reynold number.

If R < 2000, the flow of liquid is streamline or laminar. If R > 3000, the flow is turbulent. If R lies between 2000 and 3000, the flow is unstable and may change from streamline flow to turbulent flow.

EQUATION OF CONTINUITY

The equation of continuity expresses the law of conservation of mass in fluid dynamics.



$a_1 v_1 = a_2 v_2$

In general **av = constant** . This is called equation of continuity and states that as the area of cross section of the tube of flow becomes larger, the liquid's (fluid) speed becomes smaller and vice-versa.

Illustrations -

(i) Velocity of liquid is greater in the narrow tube as compared to the velocity of the liquid in a broader tube.

(ii) Deep waters run slow can be explained from the equation of continuity i.e., av = constant. Where water is deep the area of cross section increases hence velocity decreases.

ENERGY OF A LIQUID

A liquid can posses three types of energies :

(i) Kinetic energy :

The energy possessed by a liquid due to its motion is called kinetic energy. The kinetic energy of a liquid of mass m moving with speed v is $\frac{1}{2}$ mv^2.

$$\therefore \quad \text{K.E. per unit mass} = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2.$$

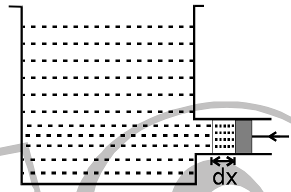
(ii) Potential energy :

The potential energy of a liquid of mass m at a height h is m g h.

$$\therefore \quad \text{P.E. per unit mass} = \frac{mgh}{m} = gh$$

(iii) Pressure energy :

The energy possessed by a liquid by virtue of its pressure is called pressure energy.
Consider a vessel fitted with piston at one side (figure). Let this vessel is filled with a liquid. Let 'A' be the area of cross section of the piston and P be the pressure experienced by the liquid.
The force acting on the piston = PA
If dx be the distance moved by the piston, then work done by the force = PA dx = PdV
where dV = Adx, volume of the liquid swept.
This work done is equal to the pressure energy of the liquid.
$\therefore$ Pressure energy of liquid in volume dV = PdV.
The mass of the liquid having volume dV = ρdV,
ρ is the density of the liquid.

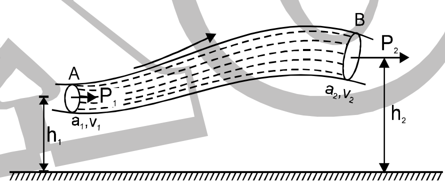


$$\therefore \text{ Pressure energy per unit mass of the liquid} = \frac{PdV}{\rho dV} = \frac{P}{\rho}.$$

BERNOULLI'S THEOREM

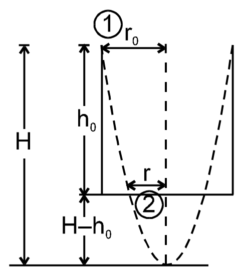
It states that the sum of pressure energy, kinetic energy and potential energy per unit mass or per unit volume or per unit weight is always constant for an ideal (i.e. incompressible and non-viscous) fluid having stream-line flow.

i.e. $\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}$.



Ex. 14 A circular cylinder of height h_0 = 10 cm and radius r_0 = 2cm is opened at the top and filled with liquid. It is rotated about its vertical axis. Determine the speed of rotation so that half the area of the bottom gets exposed. (g = 10 m/sec^2).

Sol. Area of bottom = πr_0^2
If r is radius of the exposed bottom, then



$$\pi r^2 = \frac{1}{2}\pi r_0^2 \qquad r = \frac{r_0}{\sqrt{2}}$$

Applying Bernoullis equation between points (1) and (2) -

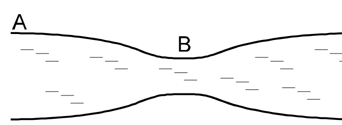
$$P_{atm} + \frac{1}{2}\rho v_1^2 - \rho gH = P_{atm} + \frac{1}{2}\rho v_2^2 - \rho g(H - h_0)$$

$$-\rho g h_0 = \frac{1}{2}\rho(v_2^2 - v_1^2) \Rightarrow 2gh_0 = [v_1^2 - v_2^2] = [w^2 r_0^2 - w^2 r^2]$$

$$r_0 = 2 \times 10^{-2}\ \text{m} \Rightarrow 2gh_0 = w^2[r_0^2 - r^2]$$

$$w = \frac{2}{r_0}\sqrt{gh} = \frac{2}{2\times 10^{-2}}\sqrt{10\times 0.1} = 100 \text{ radian / sec.}$$

Ex. 15 Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between A and B where the areas of cross-section are 30 cm^2 and 15 cm^2 respectively. Find the rate of flow of water through the tube.



Sol. Let the velocity at A = v_A and that at B = v_B.

By the equation of continuity, $\frac{v_B}{v_A} = \frac{30\,cm^2}{15\,cm^2} = 2$.

By Bernoulli's equation,

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

or, $P_A - P_B = \frac{1}{2}\rho(2v_A)^2 - \frac{1}{2}\rho v_A^2 = \frac{3}{2}\rho v_A^2$

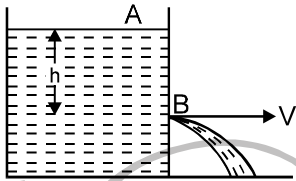
or, $600\ \frac{N}{m^2} = \frac{3}{2}\left(1000\frac{kg}{m^3}\right)v_A^2$

or, $v_A = \sqrt{0.4\ m^2/s^2} = 0.63$ m/s.

The rate of flow = (30 cm^2) (0.63 m/s) = 1800 cm^3/s.

APPLICATION OF BERNOULLI'S THEOREM

(i) Busen burner
(ii) Lift of an airfoil.
(iii) Spinning of a ball (Magnus effect)
(iv) The sprayer.
(v) A ping-pong ball in an air jet
(vi) Torricelli's theorem (speed of efflux)

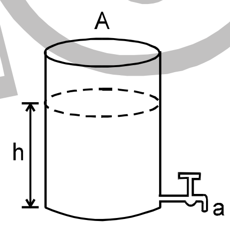


At point A, $P_1 = P$, $v_1 = 0$ and $h_1 = h$
At point B, $P_2 = P$, $v_2 = v$ (speed of efflux) and h = 0

Using Bernoulli's theorem $\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 = \frac{1}{2}v_2^2$, we have

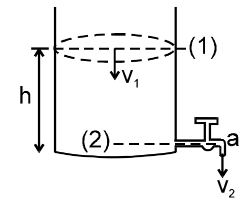
$$\frac{P}{\rho} + gh + 0 = \frac{P}{\rho} + 0 + \frac{1}{2}v^2 \quad \Rightarrow \quad \frac{1}{2}v^2 = gh \quad \text{or} \quad v = \sqrt{2gh}$$

Ex. 16 A cylindrical container of cross-section area, A is filled up with water upto height 'h'. Water may exit through a tap of cross section area 'a' in the bottom of container. Find out :



(a) Velocity of water just after opening of tap.
(b) The area of cross-section of water stream coming out of tape at depth h_0 below tap in terms of 'a' just after opening of tap.
(c) Time in which container becomes empty. (Given : $\left(\frac{a}{A}\right)^{1/2} = 0.02$, h = 20 cm, h_0 = 20 cm)

Sol. Applying Bernoulli's equation between (1) and (2) -



$$P_a + \rho gh + \frac{1}{2}\rho v_1^2 = P_a + \frac{1}{2}\rho v_2^2$$

Through continuity equation :

$$Av_1 = av_2,\ v_1 = \frac{av_2}{a} \qquad \rho gh + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$$

on solving - $v_2 = \sqrt{\frac{2gh}{1-\frac{a^2}{A^2}}} = 2$m/sec. ....(1)

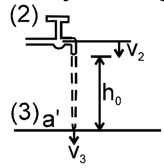
(b) Applying Bernoulli's equation between (2) and (3)

$$\frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho v_3^2$$

Through continuity equation -

Teko Classes, Maths : Suhag R. Kariya (S. R. K. Sir), Bhopal Phone : 0 903 903 7779, 0 98930 58881.

$av_2 = a' v_3 \quad \Rightarrow \quad v_3 = \frac{av_2}{a'} \quad \Rightarrow \quad \frac{1}{2}\rho v_2^{\,2} + \rho g h_0 = \frac{1}{2}\rho\left(\frac{av_2}{a'}\right)^2$



$$\frac{1}{2} \times 2 \times 2 + gh_0 = \frac{1}{2}\left(\frac{a}{a'}\right)^2 \times 2 \times 2$$

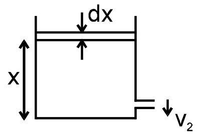
$$\left(\frac{a}{a'}\right)^2 = 1 + \frac{9.8 \times .20}{2} \Rightarrow \left(\frac{a}{a'}\right)^2 = 1.98 \quad \Rightarrow \quad a' = \frac{a}{\sqrt{1.98}}$$

(c) From (1) at any height 'h' of liquid level in container, the velocity through tap,

$$v = \sqrt{\frac{2gh}{0.98}} = \sqrt{20h}$$

we know, volume of liquid coming out of tap = decrease in volume of liquid in container.
For any small time interval 'dt'
$av_2 dt = -A \cdot dx$

$$a\sqrt{20x}\ dt = -A\ dx \quad \Rightarrow \int_0^t dt = -\frac{A}{a}\int_h^0 \frac{dx}{\sqrt{20x}}$$



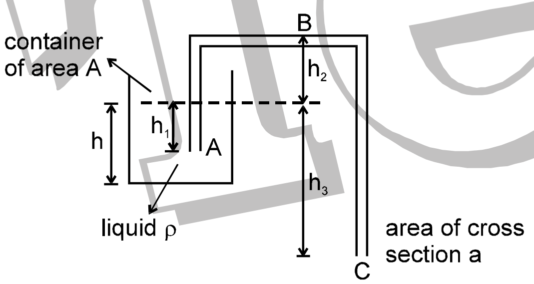
$$t = \frac{A}{a\sqrt{20}}\left[2\sqrt{x}\right]_h^0 \quad \Rightarrow \ t = \frac{A}{a\sqrt{20}}\ 2\sqrt{h}$$

$$= \frac{A}{a} \times 2 \times \sqrt{\frac{h}{20}} = \frac{2A}{a}\sqrt{\frac{0.20}{20}} = \frac{2A}{a} \times 0.1$$

Given $\left(\frac{a}{A}\right)^{1/2} = 0.02$ or $\frac{A}{a} = \frac{1}{0.0004} = 2500$

Thus t = 2 × 2500 × 0.1 = **500 second**.

Ex.17



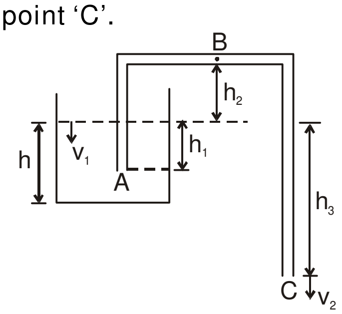
In a given arrangement **(a)** Find out velocity of water coming out of 'C'

(b) Find out pressure at A, B and C.

Sol. **(a)** Applying Bernoulli is equation between liquids surface and point 'C'.

$$p_a + \frac{1}{2}\rho v_1^2 = p_a - \rho g h_3 + \frac{1}{2}\rho v_2^2$$

through continuity equation



$$Av_1 = av_2,\ v_1 = \frac{av_2}{A} \quad \Rightarrow \frac{1}{2}\rho\frac{a^2}{A^2}v_2^2 = -\rho g h_3 + \frac{1}{2}\rho v_2^2$$

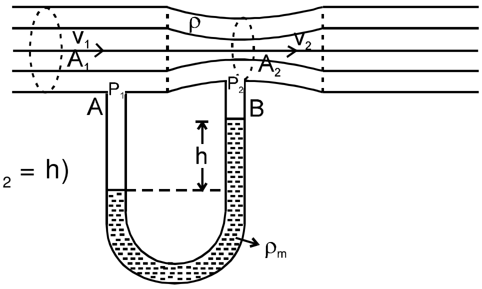
$$v_2^2 = \frac{2gh_3}{1-\frac{a^2}{A^2}},\ v_2 = \sqrt{\frac{2gh_3}{1-\frac{a^2}{A^2}}}$$

(b) Pressure at A, $p_A = p_{atm} + \rho g h_1$
Pressure at B, $p_B = p_{atm} - \rho g h_2$
Pressure at C, $p_C = p_{atm}$

(VII) Venturimeter.

It is a gauge put on a flow pipe to measure the flow of speed of a liquid (Fig). Let the liquid of density ρ be flowing through a pipe of area of cross section A_1. Let A_2 be the area of cross section at the throat and a manometer is attached as shown in the figure. Let v_1 and P_1 be the velocity of the flow and pressure at point A, v_2 and P_2 be the corresponding quantities at point B.

Using Bernoulli's theorem :



$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2 \text{, we get}$$

$$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2 \quad (\text{Since } h_1 = h_2 = h)$$

or $$(P_1 - P_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad ....(1)$$

According to continuity equation, $A_1 v_1 = A_2 v_2$

or $$v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

Substituting the value of v_2 in equation (1) we have

$$(P_1 - P_2) = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right] \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

Since $A_1 > A_2$, therefore, $P_1 > P_2$

or $$v_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]} = \frac{2A_2^2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}$$

where $(P_1 - P_2) = \rho_m gh$ and h is the difference in heights of the liquid levels in the two tubes.

$$v_1 = \sqrt{\frac{2\rho_m gh}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

The flow rate (R) i.e., the volume of the liquid flowing per second is given by $R = v_1 A_1$.

(viii) During wind storm,

The velocity of air just above the roof is large so according to Bernoulli's theorem, the pressure just above the roof is less than pressure below the roof. Due to this pressure difference an upward force acts on the roof which is blown of without damaging other parts of the house.

(ix) When a fast moving train cross a person standing near a railway trace, the person has a tendency to fall towards the train. This is because a fast moving train produces large velocity in air between person and the train and hence pressure decreases according to Bernoulli's theorem. Thus the excess pressure on the other side pushes the person towards the train.

